Two-phase Flow of Hydrogen in Horizontal Tubes

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Semirigorous equations are developed for flow of flashing liquids in pipe lines. These relations are applied to the flow of hydrogen, and computed correlations are presented for the pressure drop and vapor fraction. The calculations have been carried out as functions of the parameters: diameter, 3/8 to 4 in.; mass rate per unit area, 0.0528 to 682 lb./(sq. ft.)(sec.); pressure, 14 to 30 lb./sq. in. abs.; vapor fraction, 0.005 to 0.65; and heat leak, equivalent to 0 to 0.0224 B.t.u./(sq. ft.)(sec.) for a 3/4-in. line.

The ever-increasing use of liquid hydrogen requires a better knowledge of its behavior in transfer. Pressure drops in transfer lines can be approximated by the more simple methods of calculation for a single phase, but these methods do not suffice to describe such systems completely, as various amounts of vaporization may occur under certain transfer conditions. The further problem of critical flow may appear at conditions appreciably different from those expected from simple approximations.

THEORY

One-dimensional flow equations have been developed after the pattern of Harvey and Foust (*1*). Three basic equations—the momentum equation from Newton's second law of motion, the conservation-of-energy equation, and the continuity equation—combine to give the pressure drop and the rate of gas formation with pressure change as a function of diameter, mass rate, heat leak, and gas composition. The reduced form of the momentum equation is

$$144\frac{dP}{\rho} + \frac{u\,du}{g} + dz + \frac{2\phi^2 f(1-x)^2 u^2 \rho}{gD\rho_l}\,dL = 0 \quad (1)$$

The energy balance is given by

$$[C_l + x(C_{p_g} - C_l)]\,dT + \lambda\,dx + \frac{\mu\,du}{gJ} + \frac{dz}{J} = \frac{Q}{L}\frac{1}{\rho u A}\,dL \quad (2)$$

and the continuity equation is

$$du = \frac{u\rho}{p\rho_l}\left(\frac{10.71}{M}T\rho_l - p\right)dx - \frac{10.71}{MpT}u\rho x(K - T)\,dT \quad (3)$$

These equations are derived in detail in the appendixes of reference *1*. The fourth term in the momentum equation embodies the correlation of Lockhart and Martinelli (*2*) for two-phase flow, which is based upon data from experiments with air and various liquids as components of the flow system. The extension of the correlation to a boiling liquid was made by Harvey and Foust (*1*), and the broad assumption that such a correlation applies to the flow of liquid hydrogen is the basis of this paper.

The following assumptions are made in the derivation of the preceding equations in accordance with Harvey and Foust:

1. That there is no radial variation of pressure, temperature, velocity, quality, density, and specific heat.*
2. That the relative velocity between the vapor and liquid is zero.*
3. That the vapor and liquid are in equilibrium. This is expressed mathematically by the vapor-pressure relationship of the Clausius-Clapeyron equation

$$\frac{dp}{p} = \frac{K\,dT}{T^2} \quad (4)$$

and by the experimental data of Grilly (*3*)

$$\log\frac{760}{14.7}p = 5.5567 - \frac{54.650}{T}\frac{9}{5} \quad (5)$$

for hydrogen where the second-order correction term has been dropped.

4. That the pressure drop of a two-phase mixture can be predicted from the correlation of Lockhart and Martinelli (*2*). This correlation is expressed as

$$\left(\frac{dp}{dL}\right)_{TPF} = \phi_{ttl}^{\,2}\left(\frac{dp}{dL}\right)_l \quad (6)$$

where the liquid pressure drop is the common expression

$$\left(\frac{dp}{dL}\right)_l = \frac{f u_l^{\,2}\rho_l}{288 g R_h} \quad (7)$$

and ϕ is a function of the liquid and gas mass rates, densities, and viscosities. The number in the denominator of Equation (7) includes the conversion factor from square feet to square inches. Lockhart and Martinelli have plotted ϕ vs. X for their experimental data where the latter is defined as

$$\mathrm{X} = \left(\frac{w_l}{w_g}\right)^{0.9}\left(\frac{\rho_g}{\rho_l}\right)^{0.5}\left(\frac{\mu_l}{\mu_g}\right)^{0.1} \quad (8a)$$

$$= \left(\frac{1-x}{x}\right)^{0.9}\left(\frac{pM}{10.71T\rho_l}\right)^{0.5}\left(\frac{\mu_l}{\mu_g}\right)^{0.1} \quad (8b)$$

The velocity of the liquid is defined as

$$u_l = (1-x)\frac{u\rho}{\rho_l} \quad (9)$$

*It is recognized that these assumptions are valid only in the limit for "froth" flow which exhibits a high degree of mixing and subdivision. This implies an expected increase in reliability of the correlations at the lower quality values.

and combination of (6), (7), and (9) where $R_h = D/4$ gives

$$\left(\frac{dp}{dL}\right)_{TPF} = \frac{\phi^2 f(1-x)^2 u^2 \rho^2}{72 g D \rho_l} \quad (10)$$

5. That the hydrogen gas behaves as a perfect gas.

$$\frac{p}{\rho_g} = \frac{10.71T}{M} \quad (11)$$

6. That the heat leak is uniform along the transfer tube.
7. That the average density is given by

$$\frac{1}{\rho} = \frac{1-x}{\rho_l} + \frac{x}{\rho_g} \quad (12a)$$

and by substitution of (11) into (12*a*)

$$\frac{1}{\rho} = \frac{1-x}{\rho_l} + \frac{10.71Tx}{Mp} \quad (12b)$$

The friction factor (*4*) used in the preceding equations is defined for turbulent flow as

$$f = 0.046/Re_l^{\,0.2} \quad (13)$$

where the Reynolds number is given by

$$Re = \left(\frac{Du\rho}{\mu}\right)_l \quad (14)$$

for $5000 \leq Re \leq 200{,}000$.

Solution of Equations (1) to (4) by simple substitution and rearrangement yields the expressions shown as Equations (15) and (16) for reciprocal pressure drop and for rate of change of quality with pressure for zero change in potential energy.

For purposes of numerical calculation certain of the physical properties—liquid density, specific heats, latent heat of vaporization, and viscosities—have been assumed constant. Equations (15) and (16) have been further reduced into terms of p and ρ_l by substitution of $u = w/A\rho$ with equation (12*b*) and by substitution for T using equation (5). The area, A, in turn, is expressed in terms of diameter. The final equations after the substitutions indicated above are as shown in (17) and (18).

The ϕ term is obtained from the plot of Lockhart and Martinelli and for computational purposes was fit by the following equations:

$$\phi^2 = 13.054\mathrm{X}^{-1.36} + 2.5166 \qquad 0 < \mathrm{X} < 0.9$$

$$\phi^2 = 15.843\mathrm{X}^{-0.659} \qquad 0.9 < \mathrm{X} < 26$$

Two supplementary equations for the reciprocal pressure drop for the single

$$\left(\frac{dL}{dp}\right)_{TPF} = -\frac{\dfrac{144\lambda}{\rho} + \dfrac{u^2\rho}{gJp\rho_l}\left(\dfrac{10.71}{M}T\rho_l - p\right)\left[\dfrac{144}{\rho} - [C_l + x(C_{p_g} - C_l)]\dfrac{JT^2 1.986}{\lambda Mp}\right] - \dfrac{21.27u^2\rho xT}{M^2gp^2}\left(\dfrac{\lambda M}{1.986} - T\right)}{\dfrac{u^2\rho}{gJp\rho_l}\left(\dfrac{10.71}{M}T\rho_l - p\right)\left[\dfrac{2\phi^2 f(1-x)^2u^2\rho}{gD\rho_l} + \dfrac{Q}{L}\dfrac{J}{\rho uA}\right] + \dfrac{2\phi^2\lambda f(1-x)^2u^2\rho}{gD\rho_l}} \tag{15}$$

and

$$\frac{dx}{dp} = -\frac{\dfrac{Q}{L}\dfrac{144}{\rho} - \dfrac{21.27u^2\rho xT\left(\dfrac{\lambda M}{1.986} - T\right)}{\lambda M^2gp^2}\dfrac{Q}{L} + \dfrac{3.972\phi^2 f(1-x)^2u^3\rho^2A}{\lambda MgD\rho_l}\left[[C_l + x(C_{p_g} - C_l)]\dfrac{T^2}{p} - \dfrac{10.71u^2\rho xT}{gJMp^2}\left(\dfrac{\lambda M}{1.986} - T\right)\right]}{\dfrac{Q}{L}\dfrac{u^2\rho}{gp\rho_l}\left(\dfrac{10.71}{M}T\rho_l - p\right) + \dfrac{2\phi^2 f(1-x)^2u^3\rho^2A}{gD\rho_l}\left[\lambda + \dfrac{u^2\rho}{gJp\rho_l}\left(\dfrac{10.71}{M}T\rho_l - p\right)\right]} \tag{16}$$

$$\left(\frac{dL}{dP}\right)_{TPF} = -\frac{6335 - 3.33(10^{-6})\left(\dfrac{w}{D^2p}\right)^2\left(\dfrac{2323}{\log p - 3.844} + p\right)\left\{32.7\left[p(1-x) - \dfrac{2323x}{\log p - 3.844}\right] - \dfrac{86{,}700 + x8{,}860}{(\log p - 3.844)^2}\right\} + \left(\dfrac{w}{D^2p}\right)^2\dfrac{x}{\log p - 3.844}\left(1160 + \dfrac{587}{\log p - 3.844}\right)}{2.22(10^{-5})\phi^2\dfrac{[(1-x)w]^{1.8}}{D^{4.8}}\left\{194 - 3.33(10^{-6})\left(\dfrac{w}{D^2p}\right)^2\left[(1-x)p - \dfrac{2323x}{\log p - 3.844}\right]\left[\dfrac{2323}{\log p - 3.844} + p\right]\right\} - 2.59(10^{-3})\left(\dfrac{p}{w}\dfrac{Q}{L}\right)\left(\dfrac{w}{D^2p}\right)^2\left(\dfrac{2323}{\log p - 3.844} + p\right)} \tag{17}$$

$$\frac{dx}{dp} = \frac{144\dfrac{Q}{L} + \dfrac{Q}{L}\left(\dfrac{w}{D^2p}\right)^2 x\left(195.3 + \dfrac{98.37}{\log p - 3.844}\right)\dfrac{0.136}{\log p - 3.844} + \dfrac{\phi^2[(1-x)w]^{1.8}w}{D^{4.8}}\left\{\dfrac{(10.9 + 1.11x)10^{-3}}{p(\log p - 3.844)^2} + \dfrac{3.87(10^{-9})}{p(\log p - 3.844)}\left(\dfrac{w}{D^2p}\right)^2 x\left[(1-x)p - \dfrac{2323x}{\log p - 3.844}\right]\left[195.3 + \dfrac{98.37}{\log p - 3.844}\right]\right\}}{0.0114\dfrac{Q}{L}\left(\dfrac{w}{D^2p}\right)^2 p\left(\dfrac{2323}{\log p - 3.844} + p\right) - \dfrac{\phi^2[(1-x)w]^{1.8}w}{D^{4.8}}\left\{1.90(10^{-2}) - 3.26(10^{-10})\left(\dfrac{w}{D^2p}\right)^2\left[(1-x)p - \dfrac{2323x}{\log p - 3.844}\right]\left(\dfrac{2323}{\log p - 3.844} + p\right)\right\}} \tag{18}$$

phases, liquid and gaseous, for hydrogen are as follows:

$$\left(\frac{dL}{dp}\right)_l = 1.47(10^6)\frac{D^{4.8}}{w^{1.8}} \tag{19}$$

from Equations (7) and (13), and

$$\left(\frac{dL}{dp}\right)_g = 1041p(3.844 - \log p)\frac{D^{4.8}}{w^{1.8}} \tag{20}$$

from Equations (5 and 11) and (7 and 13) based on the gas.

The numerical results obtained by use of these equations have been reduced to eighteen graphs.*

APPLICATION OF THEORY

The problem of two-phase flow of hydrogen in horizontal tubes has been treated theoretically above. The following mass rates: w = 0.0044, 0.090, 0.180, 0.265 and 0.420 lb./sec.; diameters: 0.028, 0.0625, 0.1666, and 0.3333 ft.; and vapor fractions: 0.005 to 0.65 quality were selected for hydrogen transfer. Each individual mass rate was considered separately with each diameter parameter. These arbitrarily chosen sets of parameters were considered in turn in combination with heat leaks equivalent to the following for the 0.0625-ft. diameter line: Q/L = 0, $3.6(10)^{-4}$, and $4.4(10)^{-3}$ B.t.u./(ft.)(sec.). [These correspond to heat leaks per unit area of 0, $1.83(10)^{-3}$, and $2.24(10)^{-2}$ B.t.u./(sq. ft.)(sec.).] Heat leaks for the other transfer lines were obtained from these by multiplying by the ratio of the diameters. In practice, liquid hydrogen is transferred through vacuum-jacketed tubes with a minimum of conduction losses; hence the heat leaks are relatively small.

The two simultaneous equations [(17) and (18)] were solved together with the integral equation

$$\Delta L = \int_{p_1}^{p_2}\frac{dL}{dp}\,dp \tag{21}$$

by means of an I.B.M. 701 computation. The lower limit on vapor composition was arbitrarily chosen as 0.005 quality instead of zero as the X term of Lockhart and Martinelli goes to infinity as x goes to zero. [See Equation (8b).] Pressure decrements of 0.250 lb./sq. in. starting at initial inlet pressures of 30, 24, and 16 lb./sq. in. abs. were used for machine calculations. The calculation was terminated at x = 0.65 (as it became necessary to carry the ϕ_{ltt} vs. X plot of Lockhart and Martinelli unreasonably far for most cases), at p = 14, or at either dL/dp or dx/dp = 0, whichever occurred first during the course of the I.B.M. run.

The results of the I.B.M. calculations were reduced to graphs of length of horizontal transfer tube ΔL vs. pressure,

*Supplementary material has been deposited as document 3732 with the American Documentation Institute Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for \$2.50 for photoprints or \$1.75 for 35-mm. microfilm.

TABLE 1.

Diameter, ft.	Mass rate, lb./sec.	Heat leak, B.t.u./ft.(sec.)
Critical flow occurred with ΔL extremely small		
0.028	0.265	0, $1.6(10)^{-4}$, $1.98(10)^{-3}$
0.028	0.42	0, $1.6(10)^{-4}$, $1.98(10)^{-3}$
Extremely low dx/dp and dp/dL leading to lengths of transfer tube in gross excess of 3,000 ft. before the machine computation was terminated as stipulated in the text. Any practical length of transfer line specified would lead to a negligibly small pressure drop.		
0.0625	0.0044	0, $3.6(10)^{-4}$, $4.4(10)^{-3}$
0.1666	0.0044	0, $9.6(10)^{-4}$, $1.17(10)^{-2}$
0.1666	0.090	0, $9.6(10)^{-4}$, $1.17(10)^{-2}$
0.1666	0.180	0, $9.6(10)^{-4}$
0.333	0.0044, 0.090, 0.18, 0.265, 0.42	0, $1.92(10)^{-3}$, $2.34(10)^{-2}$
0.333	0.0044, 0.090, 0.18, 0.265, 0.42	0, $4.8(10)^{-3}$, $5.9(10)^{-2}$

with diameter, mass flow, and heat leak held constant. Typical plots are shown in Figures 1 and 2 with constant-quality lines indicated. These quality lines were most often drawn as straight lines, as the actual curvature was negligible. Figure 2 shows a critical flow boundary encountered with the mathematically computed pressure drop approaching infinity in the limit. In some cases, where several heat leaks for a given mass flow and tube diameter contributed in an almost identical manner to the pressure-drop and quality change (the mass flow effect was most significant), the data were consolidated onto a single graph. Despite the numerous combinations of diameters, mass flow rates, and heat leaks with pressure and quality as variables to determine a length of transfer tube, only eighteen graphs were developed. The remaining data were not reduced to graphs on the basis shown in Table 1, which is self-explanatory. The use of the graphs is simple, with a point on the abscissa ($x = 0.005$) representing the inlet to the transfer line. The points along the ΔL vs. pressure curves give directly the pressure from the abscissa, the corresponding length of transfer line from the inlet to that point from the ordinate, and the quality from the parametric cross plot of weight fraction.

CRITICAL FLOW

Schweppe and Foust (*5*) have derived equations for critical flow and have indicated that when either dL/dp or dx/dp approach zero the conditions for critical flow are satisfied. The first is shown in Figure 2 and the second criterion was not encountered in these calculations. Conditions which would lead to a limiting flow situation with dx/dp near zero were indicated to exist for the very low mass flow rates with considerably higher heat leaks than those treated herein.

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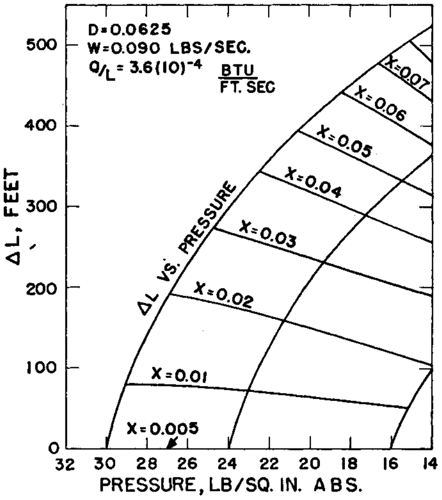

Fig. 1. Length of transfer tube vs. pressure with constant quality lines.

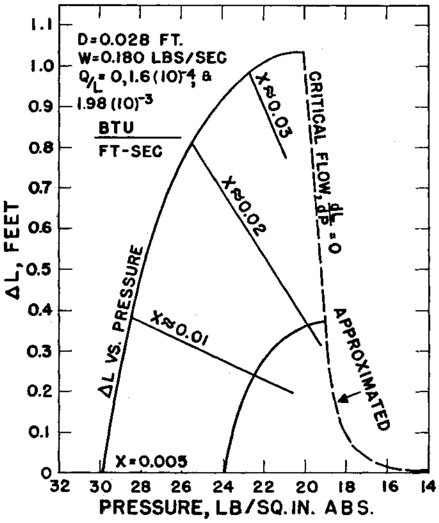

Fig. 2. Length of transfer tube vs. pressure showing the critical flow boundary.

NOTATION

Equations (17) to (20) are in English units. Some conversion factors are included in the Notation for convenience in reducing data to metric units, which are in common use in the cryogenic laboratories of the nation. The logarithms of the equations are to the base 10.

A = cross-sectional area of transfer tube, sq. ft.

C_{p_g} = specific heat at constant pressure = 2.484 B.t.u./(lb.)(°R.) = 4.968 cal./(mole)°K.)

C_l = specific heat of liquid = 2.25 B.t.u./(lb.)(°R.) = 4.50 cal./(mole)(°K.)

D = inside tube diameter, ft.

f = friction factor.

$g = g_c$ = conversion factor = 32.2 (lb.-mass)(ft.)/(lb.-force)(sec.^{2})

J = mechanical equivalent of heat = 778 ft.-lb.

K = constant in the Clausius-Clapeyron equation, $\lambda M/1.986$

L = length of transfer tube, ft.

M = molecular weight, lb./mole.

Q/L = heat leak per unit length of tube; B.t.u./(ft.)(sec.) = 8.268 cal./(cm.)(sec.)

R = universal gas constant = 1.986 cal./(°K.)(mole) = 10.71 (lb.)(cu.ft.)/(sq.in.)(°R./mole)

R_H = hydraulic radius, A/P_w, ft.

p = absolute pressure, lb./sq. in. abs.

P_w = wetted perimeter, ft.

T = temperature, °R.

u = average velocity based upon mean two phase density, ft./sec.

w = mass flow rate; lb./sec. = $1.631(10^6)$ g./hr.

x = quality (or weight fraction gas)

z = elevation head, ft.

ρ = density of the two-phase system; lb./cu. ft. = 0.0160 g./cc.

ρ_l = density of liquid = 4.41 lb./cu. ft. = 0.0705 g./cc.

ϕ, X = parameters derived by Lockhart and Martinelli (*2*)

ϕ_{ltt} = parameter based on pressure drop for liquid flowing alone, both phases turbulent

λ = latent heat of vaporization = 194 B.t.u./lb. = 215.8 cal./mole H_2

μ_g = viscosity of gaseous H_2 = $7.40(10^{-7})$ lb./(ft.)(sec.) = 11 (10^{-6}) poises

μ_l = viscosity of liquid H_2 = 8.75 (10^{-6}) lb./(ft.)(sec.) = 130 (10^{-6}) poises

Subscripts

g = gas phase

l = liquid phase

TPF = combined two-phase flow system

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